

Multi-(super)graviton theory on topologically non-trivial backgrounds

Guido Cognola ^{*,a}, Emilio Elizalde ^{†,b,c} and Sergio Zerbini ^{‡,a}

^aDipartimento di Fisica, Università di Trento
and Istituto Nazionale di Fisica Nucleare,
Gruppo Collegato di Trento, Italia

^bConsejo Superior de Investigaciones Científicas
Instituto de Ciencias del Espacio (ICE/CSIC)
Campus UAB, Facultat Ciències, Torre C5-Parell-2a planta,
08193 Bellaterra (Barcelona), Spain

^c Institut d'Estudis Espacials de Catalunya (IEEC),
Edifici Nexus, Gran Capità 2-4, 08034 Barcelona, Spain

Abstract: *It is shown that in some multi-supergraviton models, the contributions to the effective potential due to a non-trivial topology can be positive, giving rise in this way to a positive cosmological constant, as demanded by cosmological observations.*

1 Introduction

Renewed interest in the study of multi-graviton theories [1] owes, in particular, to the fact that these formulations resemble higher-dimensional gravities in the presence of discrete dimensions. These classes of discretized Kaluza-Klein theories are now in fact under the focus of attention due to their primary importance for the realization of the dimensional deconstruction program [2, 3]. Moreover, multigravitons can be also related with discretized brane-world models [4].

In spite of the absence of a consistent interaction among the gravitons, one can think of possible couplings in the theory space. In particular, in a recent paper [5], a multi-graviton theory with nearest-neighbor couplings in the theory space has been proposed. As a result, a discrete mass spectrum appears. The theory seems to be equivalent to Kaluza-Klein gravity with a discretized dimension.

In a previous paper concerning multi-graviton theory [6], we have shown by means of an explicit example, namely a discretized Randall-Sundrum (RS) brane-world [7], that the induced cosmological constant becomes positive provided the number of massive gravitons is sufficiently large.

In the present paper, we would like to show that an alternative mechanisms can also give rise to positive contributions to the cosmological constant. In particular we shall consider a multi-supergraviton example with few gravitons, in a manifold (bulk) with non trivial topology. We shall show that in such a model a positive cosmological constant Λ can be generated, due to the presence of positive topological contributions. Moreover, by a suitable choice of the topological parameters, the number obtained for Λ can reach a value perfectly in accordance with result obtained from recent cosmological observations [8].

*e-mail: cognola@science.unitn.it

†e-mail: elizalde@ieec.uab.es

‡e-mail: zerbini@science.unitn.it

2 The multi-graviton and multi-supergraviton models

2.1 The graviton model

We start by considering the Lagrangian for the spin-two field $h_{\mu\nu}$ with mass m

$$\begin{aligned}\mathcal{L}_m &= \mathcal{L}_0 - \frac{m^2}{2} (h_{\mu\nu}h^{\mu\nu} - h^2) - 2(mA^\mu + \partial^\mu\varphi)(\partial^\nu h_{\mu\nu} - \partial_\mu h) \\ &\quad - \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu),\end{aligned}\quad (2.1)$$

where \mathcal{L}_0 is the Lagrangian of the massless spin-two field (graviton) $h_{\mu\nu}$ ($h \equiv h^\mu{}_\mu$)

$$\mathcal{L}_0 = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\lambda h^\lambda{}_\mu\partial_\nu h^{\mu\nu} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h, \quad (2.2)$$

while A_μ and φ are Stückelberg fields [9].

The multi-graviton model is defined by taking N -copies of (2.1) with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$ and φ_n . Here, we propose a theory defined by a Lagrangian which is taken to be a generalization of the one in [5]. It reads

$$\begin{aligned}\mathcal{L} &= \sum_{n=0}^{N-1} \left[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_n^\lambda{}_\mu\partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n\partial^\lambda h_n \right. \\ &\quad - \frac{1}{2}(m^2\Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2) - 2(m\Delta^\dagger A_n^\mu + \partial^\mu\varphi_n)(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n) \\ &\quad \left. - \frac{1}{2}(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu})(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu) \right].\end{aligned}\quad (2.3)$$

The Δ and Δ^\dagger are difference operators, which operate on the indices n as

$$\Delta\phi_n \equiv \sum_{k=0}^{N-1} a_k\phi_{n+k}, \quad \Delta^\dagger\phi_n \equiv \sum_{k=0}^{N-1} a_k\phi_{n-k}, \quad \sum_{k=0}^{N-1} a_k = 0, \quad (2.4)$$

where the a_k are N constants and the N variables ϕ_n can be identified with periodic fields on a lattice with N sites if the *periodic boundary conditions* $\phi_{n+N} = \phi_n$ are imposed. The latter condition in (2.4) assures that Δ becomes the usual differentiation operator in a properly defined continuum limit.

The eigenvalues and eigenvectors for Δ are given by⁴

$$\Delta\phi_n^p = i\mu_p\phi_n^p, \quad i\mu_p = \sum_{n=0}^{N-1} a_n e^{2\pi i np/N}, \quad (2.5)$$

$$\phi_n^p = \frac{e^{2\pi i np/N}}{\sqrt{N}}. \quad p = 0, 1, 2, 3, \dots \quad (2.6)$$

By using (2.4) in the latter equation and assuming a_n to be real one gets the relations

$$\mu_0 = 0, \quad \mu_p = -\mu_{N-p}, \quad \mu_{N-p} = -\mu_p^*, \quad (2.7)$$

⁴Please note that here we use a different notation with respect to one used in Refs. [6] and [11]. In fact, in order to avoid confusion with masses, we have replaced the eigenvalue m with μ and the index M with p .

which, for any fixed N , permit to obtain the whole spectrum of the theory.

Then we see that the Lagrangian (2.3) describes a massless graviton and $N - 1$ massive gravitons, with masses $M_p = |\mu_p|$ ($p = 1, 2, \dots, N - 1$). It must be pointed out that the massive gravitons always appear in pairs which share a common mass and, moreover, the complex mass parameter μ_p can be arbitrarily chosen, just by properly selecting the coefficients a_k in (2.5) [6]. As discussed in [5], the multigraviton model can be regarded as corresponding to a Kaluza-Klein theory where the extra dimension lives in a lattice.

As a specific example, we now consider the two-brane Randall-Sundrum (RS) model [7] (for a recent review see [10]). In this model, the masses of the Kaluza-Klein modes are given by

$$M_p = \frac{\pi p}{z_c}, \quad z_c = l \left(e^{\pi r_c/l} - 1 \right), \quad p = 0, 1, 2, \dots \quad (2.8)$$

where l is the length parameter of the five-dimensional AdS space and πr_c the geodesic distance between the two branes.

Motivated by this last equation (2.8), we consider an $N = 2N' + 1$ graviton model, with the graviton masses being given by

$$\mu_p = \begin{cases} \frac{\pi p}{z_c}, & p = 0, 1, \dots, N', \\ -\frac{\pi(N-p)}{z_c}, & p = N' + 1, N' + 2, \dots, N - 1 = 2N'. \end{cases}$$

Those are solutions of Eq. (2.5), with the choice $a_0 = 0$ and, for any $n \geq 1$,

$$\begin{aligned} a_n &= -\frac{2\pi}{(2N' + 1)z_c} \operatorname{Im} \left\{ \frac{\left(1 - e^{-i\frac{2\pi N' n}{2N'+1}} \right) e^{-i\frac{2\pi n}{2N'+1}}}{1 - e^{-i\frac{2\pi n}{2N'+1}}} \right\} \\ &= -\frac{(-1)^n 2\pi}{N z_c} \frac{\sin^2 \left(\frac{\pi n N'}{N} \right)}{\sin \left(\frac{\pi n}{N} \right)}. \end{aligned} \quad (2.9)$$

We see that N plays here the role of a cutoff of the Kaluza-Klein modes.

In previous models of deconstruction [3, 5], mainly nearest neighbor couplings between the sites of the lattice have been considered. As a consequence, on imposing a periodic boundary condition, the lattice then looks as a circle. Departing from this standard situation, in the model considered here we have introduced *non-nearest-neighbor* couplings among the sites. That is, a site links to a number of other ones in a rather complicated way. In this sense, the lattice in the present model is no more a simple circle but it looks more like, say, a mesh or a net. Let us assume that the sites on the lattice would correspond to points in a brane. If the codimension of the spacetime is one, the brane should be ordered, resembling the sheets of a book. One brane can only interact (directly) with the two neighboring branes. However, if the spacetime is more complicated and/or the codimension is two or more, the brane can directly interact with three or more branes, an interaction that will be perfectly described by our model corresponding to this case. For example, a site on a tetrahedron connects directly with three neighboring sites. In this way, the non-nearest-neighbor couplings we here consider may quite adequately reflect the structure of the extra dimension. In this respect our model is very general and opens a number of interesting possibilities.

2.2 The supergravity case

By using the same sort of techniques described above, the multi-graviton model can be generalized to the supergravity case, just by starting with a supergravity theory in 5-dimensions and implementing deconstruction by way of replacing the fifth spacelike dimension with a one-dimensional lattice containing N -points. A multi-supergravity model of this kind has been proposed in Ref. [11], to which the interested reader is addressed for details. Here we shall only write down the essential aspects which will be used in what follows.

In the 5-dimensional linearized supergravity theory, the number of bosonic degrees of freedom is 8, 5 due to the massless graviton and 3 due to the massless vector (gauge) field and the number of fermionic degrees of freedom is 8 too, due to the the complex Rarita-Schwinger field (4×2).

The deconstruction process now consists in replacing the fifth dimension in the action of spin two+vector+Rarita-Schwinger fields with N -points and the derivatives with respect to the corresponding variable with the operator Δ as in Eq. (2.5). In this way one gets a complicated action in 4 dimensions, similar to the one in Eq. (2.3), but with vector and fermion parts too. It contains a spin-2 field $h_{\mu\nu}$ (the graviton), but also scalar, vector and fermionic fields. More precisely, in the massless sector one has 8 degrees of freedom due to bosons (graviton (2 d.o.f.), gauge and Stückelberg vectors (2+2 d.o.f.), a Stückelberg scalar and the fifth component of the gauge field (1+1 d.o.f.) and 8 degrees of freedom due to fermions (complex Dirac and Rarita-Schwinger fields), while in the massive sector one has again 8+8 degrees of freedom, but only due to a massive graviton, vector and Rarita-Schwinger fields. As in the pure-gravity case, one has N copies of such fields and their masses —obtained by imposing periodic boundary conditions— are always given by means of Eq. (2.5), that is

$$\Delta\phi_n^p = i\mu_p\phi_n^p, \quad i\mu_p = \sum_{n=0}^{N-1} a_n e^{2\pi i n p/N}, \quad (2.10)$$

$$\phi_n^p = \phi_{n+N}^p = \frac{e^{2\pi i n p/N}}{\sqrt{N}}. \quad p = 0, 1, 2, 3, \dots \quad (2.11)$$

On the other hand, for fermion fields anti-periodic boundary conditions could also be considered. In such case one gets a different spectrum, given by means of the following equations

$$\Delta\tilde{\phi}_n^p = i\tilde{\mu}_p\tilde{\phi}_n^p, \quad i\tilde{\mu}_p = \sum_{n=0}^{N-1} a_n e^{2\pi i n (p+1/2)/N}, \quad (2.12)$$

$$\tilde{\phi}_n^p = -\tilde{\phi}_{n+N}^p = \frac{e^{2\pi i n (p+1/2)/N}}{\sqrt{N}}. \quad p = 0, 1, 2, 3, \dots \quad (2.13)$$

It has to be noted that with boundary conditions of this sort, there are no massless fermions and this is a consequence of the explicitly breakdown of global supersymmetry.

3 The induced cosmological constant

We now turn to the evaluation of the induced cosmological constant for the N - graviton and super-graviton models discussed in the previous section. To this aim —the main one in the present paper— we shall compute the one-loop effective potential using zeta-function regularization [12, 13]; needless to say, other regularization schemes could work as well. First of all, we

compute the effective potential for a free scalar field with mass M , since this corresponds to the contribution of each degree of freedom to the one-loop effective potential of our theories.

In the zeta-function regularization method, the one-loop contribution to the effective potential is given by

$$V_{eff}^{(1)} = -\frac{1}{2V}\zeta'(0|L/\mu^2) = -\frac{1}{2V}\zeta'(0|L) - \frac{1}{2V}\zeta(0|L)\log\mu^2, \quad (3.1)$$

V being the volume of the manifold and $\zeta(s|L)$ the zeta function corresponding to the Laplacian-like operator $L = -\Delta^2 + M^2$, with M a positive constant. The arbitrary parameter μ has to be introduced for dimensional reasons. It will be fixed by renormalization at the end of the process.

The manifold we are considering in the present paper is a flat one with non trivial topology of the kind $\mathcal{M} = \mathbb{R} \times T^3$. The simplest case $\mathcal{M} = \mathbb{R}^4$ has been already considered in [6, 11].

The operator L has the form

$$L = -\frac{d}{d\tau^2} + L_3, \quad L_3 = -\Delta_3 + M^2, \quad (3.2)$$

Δ_3 being the Laplace operator on T^3 . The zeta-function is expressed in terms of the heat trace via the Mellin representation. The heat traces read

$$\text{Tr } e^{-tL} = V\mathcal{K}(t|L), \quad \text{Tr } e^{-tL_3} = V_3\mathcal{K}(t|L_3), \quad \mathcal{K}(t|L) = \frac{\mathcal{K}(t|L_3)}{\sqrt{4\pi t}}. \quad (3.3)$$

As a result

$$\begin{aligned} \zeta(s|L) &= \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} \text{Tr } e^{-tL} = \frac{V}{\sqrt{4\pi}\Gamma(s)} \int_0^\infty dt t^{s-3/2} \mathcal{K}(t|L_3) \\ &= \frac{V\Gamma(s-1/2)}{\sqrt{4\pi}\Gamma(s)} \tilde{\zeta}(s-1/2|L_3), \end{aligned} \quad (3.4)$$

$\tilde{\zeta}(s-1/2|L_3)$ being the zeta-function density on T^3 and $V_3 = (2\pi r)^3$ the “volume” of the torus with “radius” r . The heat kernel and zeta function on T^3 are well known. In the Appendix A, for the reader’s convenience, we summarize some useful representations that will be used in what follows (for a review, see [13]).

Using expressions (3.4) and (A.1) one realizes that the zeta function can be written as the sum of two terms, that is

$$\zeta(s|L) = \zeta_0(s|L) + \zeta_T(s|L), \quad (3.5)$$

where ζ_0 is the same one has on \mathbb{R}^4 , namely

$$\zeta_0(s|L) = \frac{V\Gamma(s-2)M^{4-2s}}{16\pi^2\Gamma(s)} = \frac{VM^{4-2s}}{16\pi^2(s-1)(s-2)}, \quad (3.6)$$

while ζ_T represents the contribution due to the non-trivial topology, which explicitly depends on the topological parameter r . Expression (3.6) is also the leading contribution to the whole zeta function in a power series expansion for large values of M .

Recalling now (A.4), we obtain

$$\zeta_T(s|L) = \frac{V\Gamma(s-3/2)\cos\pi s M^{4-2s}}{8\pi^{5/2}\Gamma(s)} \int_1^\infty du G(Mru) (u^2-1)^{3/2-s}. \quad (3.7)$$

Observe that the topological contribution vanishes at $s = 0$, and this means that

$$\zeta(0|L) = \zeta_0(0|L) = \frac{VM^4}{32\pi^2}. \quad (3.8)$$

Using (3.1), for the one-loop effective potential we finally have

$$V_{eff}^{(1)} = \frac{M^4}{64\pi^2} \left(\log \frac{M^2}{\mu^2} - \frac{3}{2} \right) - \frac{M^4}{12\pi^2} \int_1^\infty du G(Mru) (u^2 - 1)^{3/2}. \quad (3.9)$$

It is interesting to note that for scalar fields, in the large mass case the topological contribution is always negative, and it is negligible with respect to the standard Coleman-Weinberg term.

As we have anticipated above, the parameter μ has to be fixed by a renormalization condition. To this aim, here we follow Ref. [14]. The total one-loop effective potential is of the form

$$V_{eff} = V_R(\mu) + V_{eff}^{(1)}(\mu), \quad (3.10)$$

$V_R(\mu)$ being the renormalized vacuum energy. For physical reasons, the last expression has to be independent of μ , and this means that

$$\mu \frac{dV_{eff}}{d\mu} = 0, \quad (3.11)$$

from which it follows that

$$V_R(\mu) = V_R(\mu_R) + \frac{M^4}{64\pi^2} \log \frac{\mu_R^2}{\mu^2}, \quad (3.12)$$

μ_R being the renormalization point which has to be fixed by the condition $V_R(\mu_R) = 0$. In this way, we finally get

$$V_{eff} = \frac{M^4}{64\pi^2} \left(\log \frac{M^2}{\mu_R^2} - \frac{3}{2} \right) + V_T(r), \quad (3.13)$$

$$V_T(r) = -\frac{M^4}{12\pi^2} \int_1^\infty du G(Mru) (u^2 - 1)^{3/2} = -\frac{M^2}{16\pi^4 r^2} \sum_{n \in \mathbb{Z}^3; n \neq 0} \frac{K_2(2\pi Mr |\vec{n}|)}{n^2}. \quad (3.14)$$

$V_T(r)$ represents the contribution coming from the non-trivial topology, which for scalar fields is always negative. We also note that, as a function of the topological parameter r , $V_T(r)$ can reach, in principle, any negative value. In Eq. (3.14), K_ν are the MacDonald's (or modified Bessel) functions.

Before proceeding with the computation of the induced cosmological constant corresponding to the models we have discussed in Sect. 2, we first analyse here the behavior of $V_T(r)$ as a function of r . To this aim, we consider the two different regimes $Mr \ll 1$ and $Mr \gg 1$.

For the case $Mr \ll 1$, using (A.5) in (3.14) we get

$$\begin{aligned} V_T(r) &= -\frac{M^4}{12\pi^2} \int_1^\infty du G(Mru) (u^2 - 1)^{3/2} \\ &= -\frac{1}{12\pi^2 r^4} \int_{Mr}^\infty dx G(x) (x^2 - M^2 r^2)^{3/2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{12\pi^2 r^4} \left[\int_{Mr}^1 dx \left(1 - \frac{1}{\pi^2 x^3} \right) (x^2 - M^2 r^2)^{3/2} \right. \\
&\quad \left. + \int_0^1 dx G_0(x) x^3 + \int_1^\infty dx G(x) x^3 + O(M^2 r^2) \right] \\
&= -\frac{1}{32\pi^2 r^4} \left[\sum_{n \in \mathbb{Z}^3; |\vec{n}| \neq 0} \frac{1}{\pi^4 |\vec{n}|^4} + O(M^2 r^2) \right] \sim -\frac{1}{64\pi^2 r^4} + O(M^2/r^2). \quad (3.15)
\end{aligned}$$

We thus see that in this limit the leading term does not depend on M , and that it can be arbitrarily large, with a suitable choice of the parameter r . The series in the latter equation has been computed numerically.

On the contrary, in the opposite regime, $Mr \gg 1$, using (3.14) and the asymptotic expansion for the Bessel function, we obtain

$$\begin{aligned}
V_T(r) &= -\frac{M^2}{16\pi^4 r^2} \sum_{n \in \mathbb{Z}^3; n \neq 0} \frac{K_2(2\pi Mr |\vec{n}|)}{n^2} \\
&\sim -\frac{3M^4}{32\pi^4 (Mr)^{5/2}} e^{-2\pi Mr} + \dots \quad (3.16)
\end{aligned}$$

In this limit the topological contribution could indeed be arbitrarily small. In Fig. 1 the whole behavior of the topological part of the effective potential is drawn. In order to work with dimensionless variables we have introduced the function $\tilde{V}_T(y) \equiv r^4 V_T(r)$ of the dimensionless variable $y \equiv Mr$. The graphic corresponds to the exact expression for $\tilde{V}_T(y)$, as given e.g. by the first lines of Eq. (3.15), multiplied by $3 \cdot 64\pi^2$. A very smooth transition from the behavior corresponding to $Mr \ll 1$, Eq. (3.15), to the one for $Mr \gg 1$, Eq. (3.16), is revealed. In Fig. 2, the corresponding graphic of the full effective potential, Eq. (3.13), is drawn, again as a function of y and setting $\mu_R r = 1$.

At this point, the effective potential –and, as a consequence, the induced cosmological constant for the models we are interested in– can be obtained by adding up several contributions of the kind (3.13).

3.1 The multi-graviton model

We start with the explicit example of multi-graviton model given by (2.9), in which there is a single massless graviton and $(N-1)/2$ couples of massive gravitons, with masses given by

$$M_0 = |\mu_0| = 0, \quad M_p = |\mu_p| = \frac{\pi p}{z_c}, \quad p = 1, 2, \dots, \frac{N-1}{2}. \quad (3.17)$$

On the manifold $\mathcal{M} = \mathbb{R} \times T^3$, the massless graviton gives no contribution to the effective potential, while it does appear explicitly on manifolds with a non-vanishing curvature. Since the massive gravitons always show up in pairs, in order to perform the computation of the effective potential, it is sufficient to consider only one half of the whole massive spectrum. Moreover, we have to take into account that each massive graviton contributes with five scalar degrees of freedom. After these considerations have been properly taken into account, for the effective potential we get the following expression

$$V_{eff} = 10 \sum_{p=1}^{\frac{N-1}{2}} \frac{M_p^4}{64\pi^2} \left(\ln \frac{M_p^2}{\mu_R^2} - \frac{3}{2} \right)$$

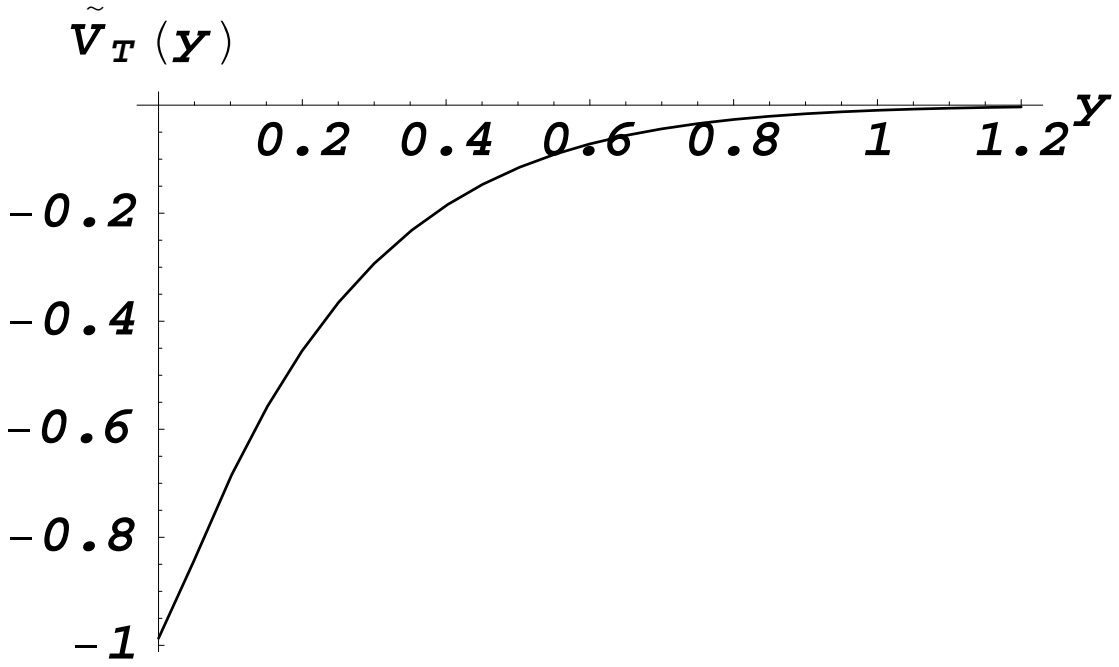


Figure 1: The exact expression for $\tilde{V}_T(y) \equiv r^4 V_T(r)$, multiplied by $3 \cdot 64\pi^2$, as a function of $y \equiv Mr$.

$$-10 \sum_{p=1}^{\frac{N-1}{2}} \frac{M_p^4}{12\pi^2} \int_1^\infty du G(M_p r u) (u^2 - 1)^{3/2}. \quad (3.18)$$

One can see that, as for the non-compact flat case (see Ref. [6] for details), in order to have a (small) positive induced cosmological constant one has to consider a large value of N , that is, a huge number of massive gravitons. In this respect, the torus topology does not improve the situation. As we are now going to show, this is no longer the case for the multi-supergraviton model.

3.2 The multi-supergraviton model

Here we have to distinguish two cases: the first one corresponds to the choice of periodic boundary conditions in both the bosonic and fermionic sectors. In such situation, the degrees of freedom due to bosons exactly compensate the degrees of freedom due to fermions. Moreover, for any massive boson there is a fermion with the same mass and, since the contribution to the effective potential of any fermionic degree of freedom is opposite to the contribution of a bosonic degree of freedom, it turns out that the induced cosmological constant vanishes, independently of the mass spectrum.

In the second case, that is when anti-periodic boundary conditions are imposed in the fermionic sector, the situation changes completely, since the fermionic mass spectrum becomes really different with respect to the bosonic one. For example, by choosing $N = 3$ [11], the solutions of Eqs. (2.10) and (2.12) give

$$M_0 = 0, \quad M_1 = M_2 = m, \quad \text{for bosons}, \quad (3.19)$$

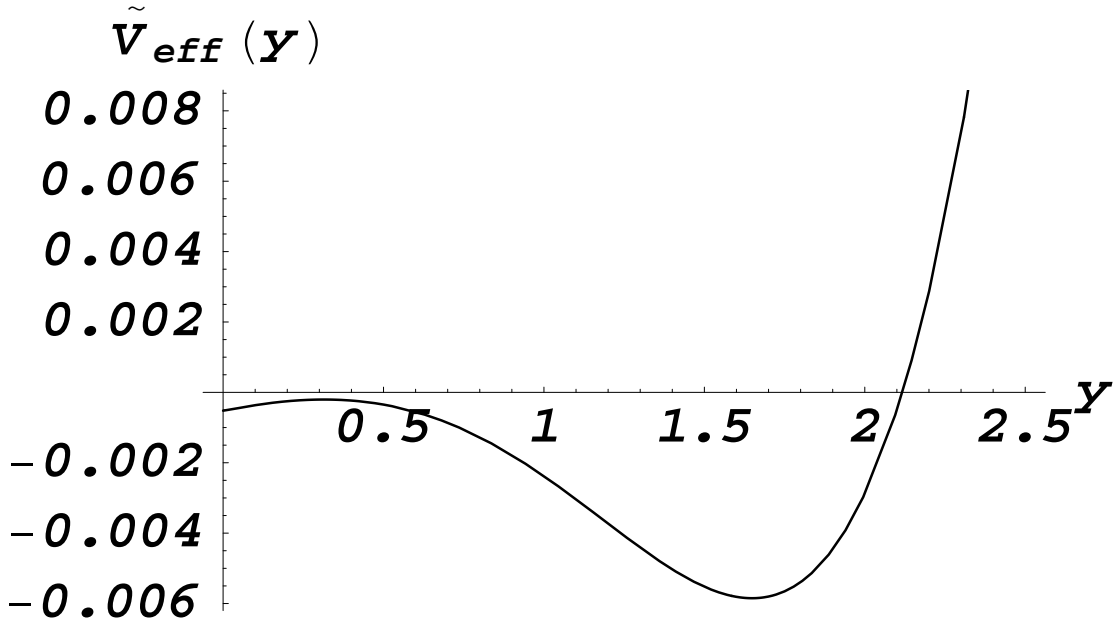


Figure 2: The exact expression for $\tilde{V}_{eff}(y) \equiv r^4 V_{eff}(r)$, Eq. (3.13), as a function of $y \equiv Mr$.

$$\tilde{M}_0 = \tilde{M}_2 = \frac{m}{\sqrt{3}}, \quad \tilde{M}_1 = \frac{2m}{\sqrt{3}}, \quad \text{for fermions,} \quad (3.20)$$

m being an arbitrary mass parameter.

By taking into account the number of degrees of freedom, the one-loop effective potential becomes, in this case

$$\begin{aligned} V_{eff} &= \frac{M_1^4}{4\pi^2} \left(\ln \frac{M_1^2}{\mu_R^2} - \frac{3}{2} \right) - \frac{4M_1^4}{3\pi^2} \int_1^\infty du G(M_1 r u) (u^2 - 1)^{3/2} \\ &\quad - \frac{\tilde{M}_0^4}{4\pi^2} \left(\ln \frac{\tilde{M}_0^2}{\mu_R^2} - \frac{3}{2} \right) + \frac{4\tilde{M}_0^4}{3\pi^2} \int_1^\infty du G(\tilde{M}_0 r u) (u^2 - 1)^{3/2} \\ &\quad - \frac{\tilde{M}_1^4}{8\pi^2} \left(\ln \frac{\tilde{M}_1^2}{\mu_R^2} - \frac{3}{2} \right) + \frac{2\tilde{M}_1^4}{3\pi^2} \int_1^\infty du G(\tilde{M}_1 r u) (u^2 - 1)^{3/2} \\ &= -\frac{m^4}{36\pi^2} \log \frac{2^{16}}{3^9} + V_T, \end{aligned} \quad (3.21)$$

V_T being the sum of all the topological contributions. As one sees, the first term on the right-hand side of the latter equation is always negative, but the whole effective potential could be positive due to the presence of the topological term. For example, in the regime $mr \ll 1$, from (3.15) one has

$$V_T \sim \frac{1}{8\pi^2 r^4} \implies V_{eff} > 0 \quad \text{for} \quad mr < \left(\frac{2}{9} \log \frac{2^{16}}{3^9} \right)^{-1/4} \sim 1.4, \quad (3.22)$$

while in the opposite regime, $mr \gg 1$, by using (3.16) one can see that the topological contribution although still positive it is negligible, and thus the effective potential remains negative.

In Fig. 3, the corresponding graphic of the full effective potential, Eq. (3.21), is drawn, again as a function of $y \equiv mr$.

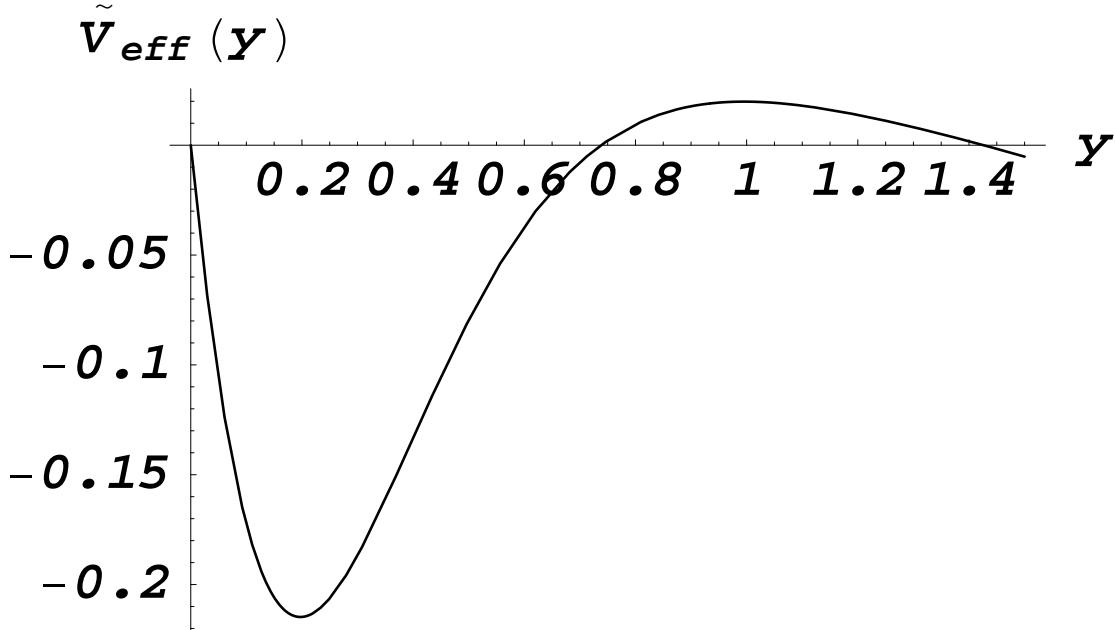


Figure 3: The exact expression for $\tilde{V}_{eff}(y) \equiv r^4 V_{eff}(r)$, Eq. (3.21), as a function of $y \equiv mr$.

4 Conclusions

In this paper, we have computed the effective potential corresponding to a multi-graviton model with supersymmetry in the case where the bulk is a flat manifold with the topology of a torus (more precisely $\mathbb{R} \times T^3$), and we have shown that the induced cosmological constant could be positive due to topological contributions.

In previous papers [6, 11] multi-graviton and multi-supergraviton models have been considered in \mathbb{R}^4 . It has been shown that in the multi-graviton model the induced cosmological constant can be positive, but only if the number of massive gravitons is sufficiently large, while in the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic boundary conditions in the fermionic sector. Note that the topological effects discussed above may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue [15].

In the case of the torus topology, the topological contributions to the effective potential have always a fixed sign, depending on the boundary conditions one imposes. In fact, they are negative for periodic fields and positive for anti-periodic fields. This means that the torus topology provides a mechanism which, in a most natural way, permits to have a positive cosmological

constant in the multi-supergravity model with anti-periodic fermions, being the value of such constant regulated by the corresponding size of the torus.¹ In this situation one can most naturally use the minimum number, $N = 3$, of copies of bosons and fermions.

We finish with the remark that it would be interesting to apply the deconstruction scheme of Ref. [6] also for the case of two latticized extra dimensions, which in the continuous limit would contain the orbifold singularity. This analysis might have a quite interesting impact on brane running coupling calculations [17].

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A Zeta function on the torus

Eigenvalues of the Laplacian on the 3-dimensional torus are of the form $\lambda_n = n^2$, $n \in \mathbb{Z}^3$, and thus the corresponding heat kernel is given by

$$\mathcal{K}(t|L_3) = \frac{e^{-tM^2}}{V_3} \sum_{n \in \mathbb{Z}^3} e^{-tn^2/r^2} = \frac{e^{-tM^2}}{(4\pi t)^{3/2}} \sum_{n \in \mathbb{Z}^3} e^{-\pi^2 n^2 r^2/t}, \quad (\text{A.1})$$

being $V_3 = (2\pi r)^3$ the “volume” of T_3 . Using the expression above, the zeta function of this Laplacian can be written as

$$\zeta(s|L_3) = \zeta_0(s|L_3) + \zeta_T(s|L_3), \quad (\text{A.2})$$

where the contribution

$$\zeta_0(s|L_3) = \frac{V_3 \Gamma(s - 3/2) M^{3/2-2s}}{(4\pi)^{3/2} \Gamma(s)}, \quad (\text{A.3})$$

comes from the $n = 0$ term and it is the same one has for \mathbb{R}^3 , while ζ_T corresponds to the contribution due to the non-trivial topology. Such term can be written in different ways, for instance, as an infinite sum of Bessel functions.

In Refs. [12] one can find many interesting results concerning zeta functions and heat kernels corresponding to operators on manifolds with constant curvature. In particular, on the torus one has the very nice representation

$$\begin{aligned} \tilde{\zeta}_T(z|L_3) &= \frac{M^{3-2z} \sin \pi z}{4\pi^2(1-z)} \int_1^\infty du G(Mru) (u^2 - 1)^{1-z} \\ &= -\frac{\pi^{z-2}}{4\Gamma(z)} \sum_{n \in \mathbb{Z}^3; n \neq 0} \left(\frac{M}{r|\vec{n}|} \right)^{3/2-z} K_{3/2-z}(2\pi Mr|\vec{n}|), \end{aligned} \quad (\text{A.4})$$

¹A more crude analysis for the pure scalar case already hinted towards this conclusion. However, the sign issue was there not easy to fix [16], the reason being now clear.

where $G(x)$ is given by

$$\begin{aligned} G(x) &= \sum_{n \in \mathbb{Z}^3; n \neq 0} e^{-2\pi|\vec{n}|x} = -1 + \frac{x}{\pi^2} \sum_{n \in \mathbb{Z}^3} \frac{1}{(n^2 + x^2)^2} \\ &= -1 + \frac{1}{\pi^2 x^3} + \frac{x}{\pi^2} G_0(x), \end{aligned} \quad (\text{A.5})$$

$G_0(x)$ being the regular function

$$G_0(x) = \sum_{n \in \mathbb{Z}^3; n \neq 0} \frac{1}{(n^2 + x^2)^2}. \quad (\text{A.6})$$

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